Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Advanced

Thursday 24 January 2008 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

N25252A

1. A parcel of mass 2.5 kg is moving in a straight line on a smooth horizontal floor. Initially the parcel is moving with speed 8 m s^{-1}. The parcel is brought to rest in a distance of 20 m by a constant horizontal force of magnitude R newtons. Modelling the parcel as a particle, find

(*a*) the kinetic energy lost by the parcel in coming to rest,

(2)

(*b*) the value of R.

(3)

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2. At time t seconds ($t \geq 0$), a particle P has position vector **p** metres, with respect to a fixed origin O, where

$$\mathbf{p} = (3t^2 - 6t + 4)\mathbf{i} + (3t^3 - 4t)\mathbf{j}.$$

Find

(*a*) the velocity of P at time t seconds,

(2)

(*b*) the value of t when P is moving parallel to the vector **i**.

(3)

When $t = 1$, the particle P receives an impulse of $(2\mathbf{i} - 6\mathbf{j})$ N s. Given that the mass of P is 0.5 kg,

(*c*) find the velocity of P immediately after the impulse.

(4)

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3. A car of mass 1000 kg is moving at a constant speed of 16 m s^{-1} up a straight road inclined at an angle θ to the horizontal. The rate of working of the engine of the car is 20 kW and the resistance to motion from non-gravitational forces is modelled as a constant force of magnitude 550 N.

(*a*) Show that $\sin \theta = \frac{1}{14}$.

(5)

When the car is travelling up the road at 16 m s^{-1}, the engine is switched off. The car comes to rest, without braking, having moved a distance y metres from the point where the engine was switched off. The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 550 N.

(*b*) Find the value of y.

(4)

4.

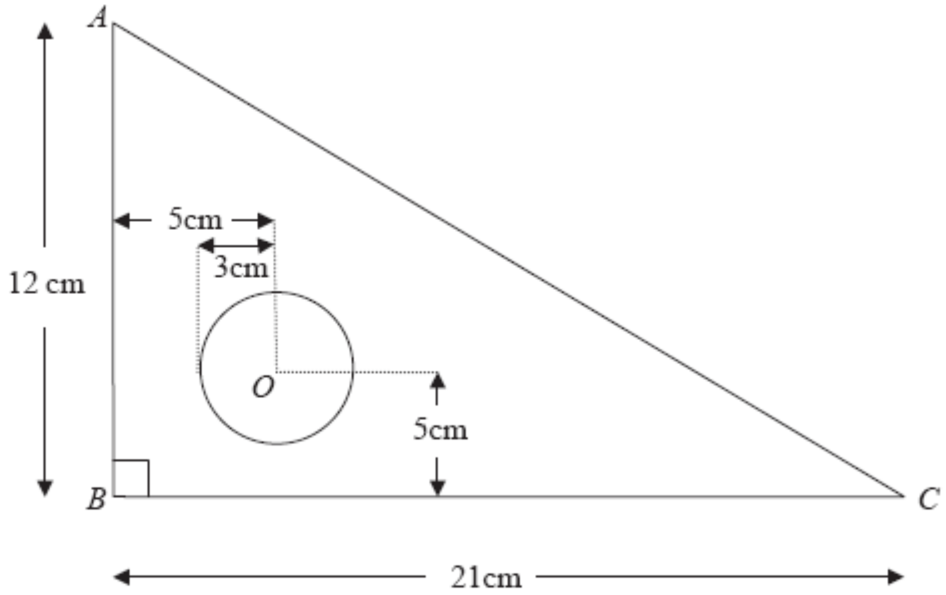

Figure 1

A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC, with $\angle ABC = 90°$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC, as shown in Figure 1.

(*a*) Find the distance of the centre of mass of S from

(i) AB,

(ii) BC.

(9)

The set square is freely suspended from C and hangs in equilibrium.

(*b*) Find, to the nearest degree, the angle between CB and the vertical.

(3)

5.

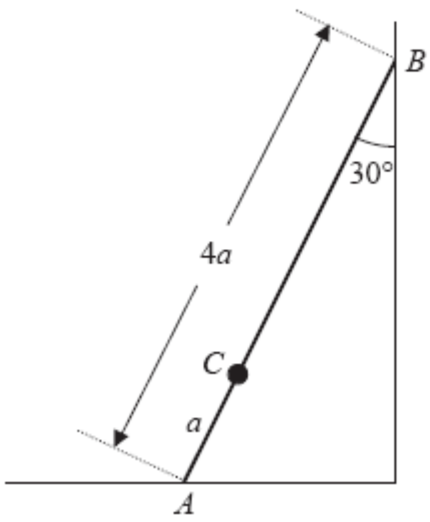

Figure 2

A ladder AB, of mass m and length $4a$, has one end A resting on rough horizontal ground. The other end B rests against a smooth vertical wall. A load of mass $3m$ is fixed on the ladder at the point C, where $AC = a$. The ladder is modelled as a uniform rod in a vertical plane perpendicular to the wall and the load is modelled as a particle. The ladder rests in limiting equilibrium making an angle of 30° with the wall, as shown in Figure 2.

Find the coefficient of friction between the ladder and the ground.

(10)

6.

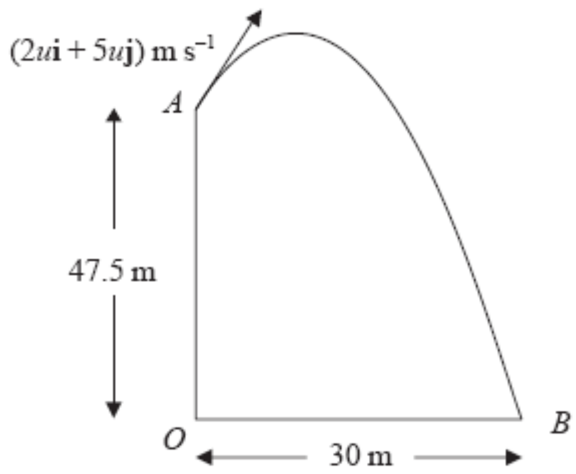

Figure 3

[*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertical.*]

A particle P is projected from the point A which has position vector $47.5\mathbf{j}$ metres with respect to a fixed origin O. The velocity of projection of P is $(2u\mathbf{i} + 5u\mathbf{j})$ m s^{-1}. The particle moves freely under gravity passing through the point B with position vector $30\mathbf{i}$ metres, as shown in Figure 3.

(*a*) Show that the time taken for P to move from A to B is 5 s.

(6)

(*b*) Find the value of u.

(2)

(*c*) Find the speed of P at B.

(5)

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7. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(*a*) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$.

(5)

(*b*) Find the total kinetic energy lost in the collision.

(5)

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(*c*) Calculate the range of values of e for which there will be a second collision between P and Q.

(7)

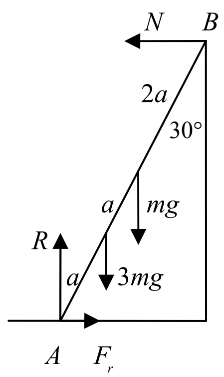
TOTAL FOR PAPER: 75 MARKS

END

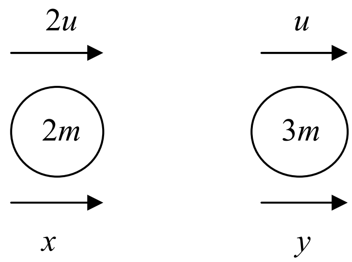
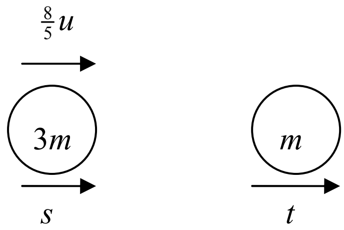
January 2008
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
1.	(a) KE lost is $\frac{1}{2}\times 2.5\times 8^2 = 80$ (J)	M1 A1 **(2)**
	(b) Work energy $80 = R\times 20$ ft their (a) $R = 4$	M1 A1 ft A1 **(3)** **[5]**
	Alternative to (b) $0^2 = 8^2 - 2\times a\times 20 \Rightarrow a = (-)1.6$ N2L $R = 2.5\times 1.6$ ft their a $= 4$	 M1 A1ft A1 **(3)**
2.	(a) $\dot{\mathbf{p}} = (6t-6)\mathbf{i} + (9t^2-4)\mathbf{j}$ $(\mathrm{m\,s^{-1}})$	M1 A1 **(2)**
	(b) $9t^2 - 4 = 0$ $t = \frac{2}{3}$	M1 DM1 A1 **(3)**
	(c) $t = 1 \Rightarrow \dot{\mathbf{p}} = 5\mathbf{j}$ ft their $\dot{p}$ (+/-) $2\mathbf{i} - 6\mathbf{j} = 0.5(\mathbf{v} - 5\mathbf{j})$ $\mathbf{v} = 4\mathbf{i} - 7\mathbf{j}$ $(\mathrm{m\,s^{-1}})$	B1ft M1 M1 A1 **(4)** **[9]**

Question Number	Scheme	Marks
3.	(a) $20000 = 16F \quad (F = 1250)$	M1 A1
	↗ $F = 550 + 1000 \times 9.8 \sin\theta$ ft their F	M1 A1ft
	Leading to $\sin\theta = \frac{1}{14}$ ✱ cso	A1 **(5)**
	(b) N2L ↗ $550 + 1000 \times 9.8 \times \sin\theta = 1000a$	
	$(550 + 1000 \times 9.8 \times \frac{1}{14} = 1000a)$	M1 A1
	or $1250 = 1000a$	
	$(a = (-)1.25)$	
	$v^2 = u^2 + 2as \Rightarrow 16^2 = 2 \times 1.25 \times y$	M1
	$y \approx 102$ accept 102.4, 100	A1 **(4)**
		[9]
	Alternative to (b)	
	Work-Energy $\frac{1}{2} \times 1000 \times 16^2 - 1000 \times 9.8 \times \frac{1}{14} y = 550y$	M1 M1 A1
	$y \approx 102$ accept 102.4, 100	A1 **(4)**
4.	(a)	
	Mass ratio: Triangle 126; Circle 9π (28.3); S $126 - 9\pi$ (97.7)	B1 B1ft
	$\bar{x}$: Triangle 7; Circle 5; S $\bar{x}$	
	$\bar{y}$: Triangle 4; Circle 5; S $\bar{y}$ 4, 7 seen	B1
	$126 \times 7 = 9\pi \times 5 + (126 - 9\pi) \times \bar{x}$ ft their table values	M1 A1ft
	$\bar{x} \approx 7.58 \; \left(\frac{882 - 45\pi}{126 - 9\pi}\right)$ awrt 7.6	A1
	$126 \times 4 = 9\pi \times 5 + (126 - 9\pi) \times \bar{y}$ ft their table values	M1 A1ft
	$\bar{y} \approx 3.71 \; \left(\frac{504 - 45\pi}{126 - 9\pi}\right)$ awrt 3.7	A1 **(9)**
	(b) $\tan\theta = \frac{\bar{y}}{21 - \bar{x}}$ ft their $\bar{x}, \bar{y}$	M1 A1ft
	$\theta \approx 15°$	A1 **(3)**
		[12]

Question Number	Scheme	Marks
5.	(a) 	
	M(A) $N \times 4a\cos 30° = 3mg \times a\sin 30° + mg \times 2a\sin 30°$	M1 A2(1,0)
	$N = \frac{5}{4}mg\tan 30°$ $\left(= \frac{5}{4\sqrt{3}}mg = 7.07\ldots m\right)$	DM1 A1
	$\rightarrow$ $F_r = N$, $\uparrow$ $R = 4mg$	B1, B1
	Using $F_r = \mu R$	B1
	$\frac{5}{4\sqrt{3}}mg = \mu R$ for their R	M1
	$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18	A1 **(10)**
		[10]
	Alternative method:	
	M(B): $mg \times 2a\sin 30 + 3mg \times 3a\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$	
	$11mga\sin 30 + F \times 4a\cos 30 = R \times 4a\sin 30$	M1A3(2,1,0)
	$\frac{11mg}{2} + F\frac{4\sqrt{3}}{2} = 2R$	DM1A1
	$\uparrow$ $R = 4mg$,	B1
	Using $F_r = \mu R$	B1
	$8\mu\sqrt{3} = \frac{5}{2}$, $\mu = \frac{5}{16\sqrt{3}}$	M1 A1

6.	(a)	$\rightarrow \quad 30 = 2ut$	B1
		$\uparrow \quad -47.5 = 5ut - 4.9t^2$	M1 A1
		$-47.5 = 75 - 4.9t^2$ eliminating u or t	DM1
		$t^2 = \dfrac{75+47.5}{4.9} (= 25)$	DM1
		$t = 5$ ✱ cso	A1 **(6)**
	(b)	$30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$	M1 A1 **(2)**
	(c)	$\uparrow \quad \dot{y} = 5u - 9.8t = -34$ M1 requires both	M1 A1
		$\rightarrow \quad \dot{x} = 2u = 6$ $\dot{x}$ and $\dot{y}$	A1
		$v^2 = 6^2 + (-34)^2$	DM1
		$v \approx 34.5 \ (\mathrm{m\,s^{-1}})$ accept 35	A1 **(5)**
			[13]
	Alternative to (c)		
		$\tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_A^2 = m \times g \times 47.5$ with $v_A^2 = 6^2 + 15^2 = 261$	M1 A(2,1,0)
		$v_B^2 = 261 + 2 \times 9.8 \times 47.5 \ (= 1192)$	DM1
		$v_B \approx 34.5 \ (\mathrm{m\,s^{-1}})$ accept 35	A1 **(5)**
	BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$		

Question Number	Scheme	Marks
7.	(a)	
		
	LM $4mu + 3mu = 2mx + 3my$	M1 A1
	NEL $y - x = \frac{1}{2}u$	B1
	Solving to $y = \frac{8}{5}u$ ∗ cso	M1 A1 **(5)**
	(b) $x = \frac{11}{10}u$ or equivalent	B1
	Energy loss $\frac{1}{2}\times 2m\left((2u)^2 - \left(\frac{11}{10}u\right)^2\right) + \frac{1}{2}\times 3m\left(u^2 - \left(\frac{8}{5}u\right)^2\right)$	M1 A(2,1,0)
	$= \frac{9}{20}mu^2$	A1 **(5)**
	(c)	
		
	LM $\frac{24}{5}mu = 3ms + mt$	M1 A1
	NEL $t - s = \frac{8}{5}eu$	B1
	Solving to $s = \frac{2}{5}u(3-e)$	M1 A1
	For a further collision $\frac{11}{10}u > \frac{2}{5}u(3-e)$	M1
	$e > \frac{1}{4}$ ignore $e \le 1$	A1 **(7)**
		[17]